7-3 Square Root Functions and Inequalities

Main Ideas

- Graph and analyze square root functions.
- Graph square root inequalities.

New Vocabulary

square root function

square root inequality

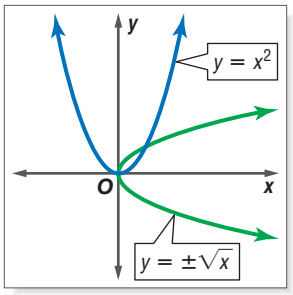
GET READY for the Lesson

The Sunshine Skyway Bridge across Tampa Bay, Florida, is supported by 21 steel cables, each 9 inches in diameter. The amount of weight that a steel cable can support is given by $w = 8d^2$, where d is the diameter of the cable in inches and w is the weight in tons.

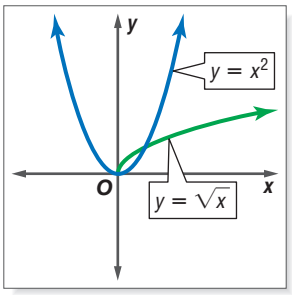
If you need to know what diameter a steel cable should have to support a given weight, you can use the equation $d = \sqrt{\frac{w}{8}}$.

Raymond Gehman/CORBIS

Square Root Functions If a function contains a square root of a variable, it is called a **square root function**. The parent function of the family of square root functions is $y = \sqrt{x}$. The inverse of a quadratic function is a square root function only if the range is restricted to nonnegative numbers.



$y = \pm\sqrt{x}$ is not a function.



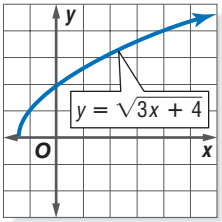
$y = \sqrt{x}$ is a function.

In order for a square root to be a real number, the radicand cannot be negative. When graphing a square root function, determine when the radicand would be negative and exclude those values from the domain.

EXAMPLE 1 Graph a Square Root Function

Graph $y = \sqrt{3x + 4}$. State the domain, range, and x- and y-intercepts.

x	y
$-\frac{4}{3}$	0
-1	1
0	2
2	3.2
4	4



Since the radicand cannot be negative, identify the domain.

$3x + 4 \geq 0$ Write the expression inside the radicand as ≥ 0.

$x \geq -\frac{4}{3}$ Solve for x.

The x-intercept is $-\frac{4}{3}$.

Make a table of values and graph the function. From the graph, you can see that the domain is $x \geq -\frac{4}{3}$, and the range is $y \geq 0$. The y-intercept is 2.

CHECK Your Progress

1. Graph $y = \sqrt{-2x + 3}$. State the domain, range, and x- and y-intercepts.

Real-World EXAMPLE 2

Real-World Link

Submarines were first used by The United States in 1776 during the Revolutionary War.

Source: www.infoplease.com

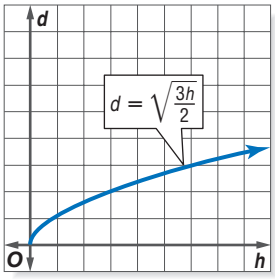
SUBMARINES **A lookout on a submarine is h feet above the surface of the water. The greatest distance d in miles that the lookout can see on a clear day is given by the square root of the quantity h multiplied by $\frac{3}{2}$.**

a. Graph the function. State the domain and range.

The function is $d = \sqrt{\frac{3h}{2}}$. Make a table of values and graph the function.

The domain is $h \geq 0$, and the range is $d \geq 0$.

h	d
0	0
2	$\sqrt{3}$ or 1.73
4	$\sqrt{6}$ or 2.45
6	$\sqrt{9}$ or 3.00
8	$\sqrt{12}$ or 3.46
10	$\sqrt{15}$ or 3.87



b. A ship is 3 miles from a submarine. How high would the submarine have to raise its periscope in order to see the ship?

$d = \sqrt{\frac{3h}{2}}$ Original equation

$3 = \sqrt{\frac{3h}{2}}$ Replace d with 3.

$9 = \frac{3h}{2}$ Square each side.

$18 = 3h$ Multiply each side by 2.

$6 = h$ Divide each side by 3.

The periscope would have to be 6 feet above the water. Check the reasonableness of this result by comparing it to the graph.

CHECK Your Progress

The speed v of a ball can be determined by the equation $v = \sqrt{\frac{2k}{m}}$, where k is the kinetic energy and m is the mass of the ball. Assume that the mass of the ball is 5 kg.

2A. Graph the function. State the domain and range.

2B. The ball is traveling 6 meters per second. What is the ball's kinetic energy in Joules?

Online Personal Tutor at algebra2.com

Like quadratic functions, graphs of square root functions can be transformed.

GRAPHING CALCULATOR LAB

Square Root Functions

You can use a TI-83/84 Plus graphing calculator to graph square root functions. Use 2nd [$\sqrt{\ }$] to enter the functions in the Y=list.

THINK AND DISCUSS

1. Graph $y = \sqrt{x}$, $y = \sqrt{x} + 1$, and $y = \sqrt{x} - 2$ in the viewing window $[-2, 8]$ by $[-4, 6]$. State the domain and range of each function and describe the similarities and differences among the graphs.
2. Graph $y = \sqrt{x}$, $y = \sqrt{2x}$, and $y = \sqrt{8x}$ in the viewing window $[0, 10]$ by $[0, 10]$. State the domain and range of each function and describe the similarities and differences among the graphs.
3. Make a conjecture about an equation that translates the graph of $y = \sqrt{x}$ to the left three units. Test your conjecture with the graphing calculator.

Square Root Inequalities A **square root inequality** is an inequality involving square roots.

EXAMPLE 3 **Graph a Square Root Inequality**

Graph $y < \sqrt{2x - 6}$.

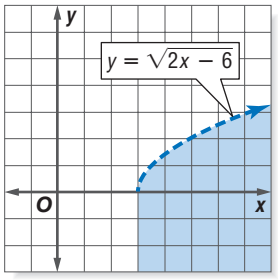
Graph $y = \sqrt{2x - 6}$. Since the boundary should not be included, the graph should be dashed.

The domain includes values for $x \geq 3$, so the graph includes $x = 3$ and values for which $x > 3$. Select a point to see if it is in the shaded region.

Test (4, 1): $1 \overset{?}{<} \sqrt{2(4) - 6}$

$1 < \sqrt{2}$ true

Shade the region that includes the point (4, 1).



Study Tip

Domain of a Square Root Inequality

The domain of a square root inequality includes only those values for which the expression under the radical sign is greater than or equal to 0.

CHECK Your Progress

3. Graph $y \geq \sqrt{x + 1}$.

CHECK Your Understanding

Example 1 (p. 398)

Graph each function. State the domain and range of the function.

1. $y = \sqrt{x} + 2$ **2.** $y = \sqrt{4x}$ **3.** $y = \sqrt{x - 1} + 3$

Example 2 (p. 398)

FIREFIGHTING For Exercises 4 and 5, use the following information.
When fighting a fire, the velocity v of water being pumped into the air is the square root of twice the product of the maximum height h and g, the acceleration due to gravity (32 ft/s^2).

4. Determine an equation that will give the maximum height of the water as a function of its velocity.

5. The Coolville Fire Department must purchase a pump that will propel water 80 feet into the air. Will a pump that is advertised to project water with a velocity of 75 ft/s meet the fire department's need? Explain.

Example 3 (p. 399)

Graph each inequality.

6. $y \leq \sqrt{x - 4} + 1$ **7.** $y > \sqrt{2x + 4}$ **8.** $y \geq \sqrt{x + 2} - 1$

Exercises

HOMEWORK HELP

For Exercises	See Examples
9–20	1
21–23	2
24–29	3

Graph each function. State the domain and range of each function.

9. $y = \sqrt{3x}$ **10.** $y = -\sqrt{5x}$ **11.** $y = -4\sqrt{x}$

12. $y = \frac{1}{2}\sqrt{x}$ **13.** $y = \sqrt{x + 2}$ **14.** $y = \sqrt{x - 7}$

15. $y = -\sqrt{2x + 1}$ **16.** $y = \sqrt{5x - 3}$ **17.** $y = \sqrt{x + 6} - 3$

18. $y = 5 - \sqrt{x + 4}$ **19.** $y = \sqrt{3x - 6} + 4$ **20.** $y = 2\sqrt{3 - 4x} + 3$

21. ROLLER COASTERS The velocity of a roller coaster as it moves down a hill is $v = \sqrt{{v_0}^2 + 64h}$, where v_0 is the initial velocity and h is the vertical drop in feet. An engineer wants a new coaster to have a velocity greater than 90 feet per second when it reaches the bottom of the hill. If the initial velocity of the coaster at the top of the hill is 10 feet per second, how high should the engineer make the hill? Is your answer reasonable?

AEROSPACE For Exercises 22 and 23, use the following information.
The force due to gravity decreases with the square of the distance from the center of Earth. As an object moves farther from Earth, its weight decreases. The radius of Earth is approximately 3960 miles. The formula relating weight and distance is $r = \sqrt{\frac{3960^2 W_E}{W_S}} - 3960$, where W_E represents the weight of a body on Earth, W_S represents its weight a certain distance from the center of Earth, and r represents the distance above Earth's surface.

22. An astronaut weighs 140 pounds on Earth and 120 pounds in space. How far is he above Earth's surface?

23. An astronaut weighs 125 pounds on Earth. What is her weight in space if she is 99 miles above the surface of Earth?

EXTRA PRACTICE
See pages 906 and 932.
Math Online
Self-Check Quiz at algebra2.com

Graph each inequality.

24. $y \leq -6\sqrt{x}$ **25.** $y < \sqrt{x + 5}$ **26.** $y > \sqrt{2x + 8}$

27. $y \geq \sqrt{5x - 8}$ **28.** $y \geq \sqrt{x - 3} + 4$ **29.** $y < \sqrt{6x - 2} + 1$

H.O.T. Problems

30. OPEN ENDED Write a square root function with a domain of $\{x \mid x \geq 2\}$.

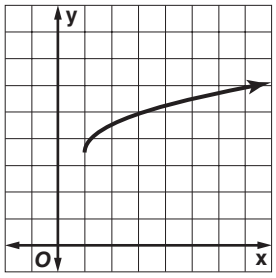
31. CHALLENGE Recall how values of a, h, and k can affect the graph of a quadratic function of the form $y = a(x - h)^2 + k$. Describe how values of a, h, and k can affect the graph of a square root function of the form $y = a\sqrt{x - h} + k$.

32. REASONING Describe the difference between the graphs of $y = \sqrt{x} - 4$ and $y = \sqrt{x - 4}$.

33. *Writing in Math* Refer to the information on page 397 to explain how square root functions can be used in bridge design. Assess the weights for which a diameter less than 1 is reasonable. Evaluate the amount of weight that the Sunshine Skyway Bridge can support.

STANDARDIZED TEST PRACTICE

34. ACT/SAT Given the graph of the square root function at the right, which must be true?



I. The domain is all real numbers.

II. The function is $y = \sqrt{x} + 3.5$.

III. The range is about $\{y \mid y \geq 3.5\}$.

A I only **C** II and III

B I, II, and III **D** III only

35. REVIEW For a game, Patricia must roll a die and draw a card from a deck of 26 cards, with each one having a letter of the alphabet on it. What is the probability that Patricia will roll an odd number and draw a letter in her name?

F $\frac{2}{3}$ **H** $\frac{1}{13}$

G $\frac{3}{26}$ **J** $\frac{1}{26}$

Spiral Review

Determine whether each pair of functions are inverse functions. (Lesson 7-2)

36. $f(x) = 3x$, $g(x) = \frac{1}{3}x$

37. $f(x) = 4x - 5$, $g(x) = \frac{1}{4}x - \frac{5}{16}$

38. $f(x) = \frac{3x + 2}{7}$, $g(x) = \frac{7x - 2}{3}$

Find $(f + g)(x)$, $(f - g)(x)$, $(f \cdot g)(x)$, and $\left(\frac{f}{g}\right)(x)$ for each $f(x)$ and $g(x)$. (Lesson 7-1)

39. $f(x) = x + 5$, $g(x) = x - 3$

40. $f(x) = 10x - 20$, $g(x) = x - 2$

41. $f(x) = 4x^2 - 9$, $g(x) = \frac{1}{2x + 3}$

42. BIOLOGY Humans blink their eyes about once every 5 seconds. How many times do humans blink their eyes in two hours? (Lesson 1-1)

GET READY for the Next Lesson

PREREQUISITE SKILL **Determine whether each number is *rational* or *irrational*.** (Lesson 1-2)

43. 4.63 **44.** π **45.** $\frac{16}{3}$ **46.** 8.333… **47.** 7.323223222…